Malthus irrelevant?

MERIT-Infonomics Research Memorandum series

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2005-010

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Malthus irrelevant?

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Abstract. We estimate and make population forecasts with Foley's (2000) model in three different ways. The population forecasts for high, middle and low-income countries are quite good and suggest that the omitted variable bias from its simplicity is small. Estimation of the model as a system shows that indeed Malthusian behaviour - defined as increasing population growth through increasing per capita income - cannot be found for any of the income groups of the Worldbank classification nor for Sub-Saharan Africa, and also not for countries with per capita income below $1200 in a panel estimate. For world aggregate data and for the low-income countries we find increasing returns to scale, but for the other groups decreasing returns (outweighed by a positive time trend except for Sub-Saharan Africa and the u1200 group). For the panel of countries with income below $1200, per capita income is stagnant for the period 1970-2002 in spite of the positive growth rates of the period 1991-2002. The time trend is as strong as the population growth in connection with decreasing returns to scale. Together with the absence of Malthusian behaviour this seems to suggest a strong role for the population growth problem as seen by David Ricardo.

1 Introduction

There is a widespread view that history was not friendly to the ideas of Thomas Malthus (see Foley 2000). According to this view his ideas were relevant from year 1000 until about 1800 but not thereafter (see van den Berg 2001). A closer look at the argument in both cases shows that the judgement is based on the use of data on the world level showing that there are positive growth rates in spite of population growth. However, the growth of per capita income may occur in the rich part of the world whereas the population growth may occur in the poorer part of the world. In this paper we want to test whether or not this judgement may turn out to be premature when we look at more disaggregated data and in particular those of low-income countries. In section 2 we report Foley's model and derive some (formally) special cases, which may be relevant for the world as a whole or for certain income groups. In section 3 we provide a single equation estimate and population forecasts based on Foley's model, which show quite realistic variants of the model in the sense that the model produces reasonable forecasts. In section 4 the model is estimated in its system form in order to get the information in regard to returns to scale and Malthusian behaviour. In section 5 we turn to van den Berg’s argument and show that for three different levels of income in terms of constant 1995 US dollars, two of which are estimates for income as of 1800, the percentage of countries below that value has not decreased since 1980. They are about stagnant in income per capita. Population growth may be one of the reasons. Moreover, the kernel density distribution for GDP per capita shows one peak at an almost constant level of per capita income of 1000 US dollar (1995) since 1960 and an increasing dispersion for countries with higher per capita income. These data all in all suggests that the group of 71 countries below $1200 (1995 dollars) was about stagnant in the period 1970-2002 in spite of positive growth rate for the period 1991-2002. Also for this group of countries we confirm Foley’s result of absence of Malthusian behaviour with decreasing returns of GDP per capita to population growth though. Section 6 summarizes and concludes that the population growth problem is still there in the way Ricardo saw it but Malthusian behaviour is not.

FIGURE 1 OVER HERE

2 Foley’s model and some special cases

Foley’s (2000) model consists of two curves with inverted U-shape (see figure 1). Income per capita is a function of population with an inverted u-shape in the lower graph of figure 1, which we call the production function. In the upward sloping (Smithian) part we have increasing returns to population growth and in the downward sloping (Malthusian) part decreasing returns to population or labour. Changes in the population are an inverted u-shape function of income per capita in the upper part of figure 1. In the upward sloping, Malthusian part population varies positively with income, and in the downward sloping Smithian part population varies negatively with income. We call it the population growth curve.

Together they work as follows. Select a value of the population. Then you find income per capita on the production function. With that income-per-capita value you go to the upper part and find the change in population, which tells you in which direction to go from your initial value. At a fertility rate of $f = 2$ population is stationary by assumption. The figure, as drawn by Foley, has four steady states. If you choose one steady-state value, imagine you are a bit to the right of it, then population is larger and output per capita is larger if you are in the increasing returns part and lower if you are in the decreasing returns part. In the upper panel we have the change in the population, which is positive if $f > 2$ ($dN > 0$) or negative if $f < 2$ ($dN < 0$). If population increases, the steady state from which you started is unstable and if it decreases it is stable. The steady states are characterized by indices *M* and *S* for 'Malthusian' and 'Smithian' as well as *U* and *S* for 'unstable' and 'stable'. Foley looks at world data, which can be plotted in the part with increasing returns and falling fertility. The steady state, which is relevant for world data then is a stable Smithian one. Once the stable Smithian steady state is reached, GDP per capita stops growing and there is a constant population. This view resembles a class of growth models (see the survey by Schneider and Ziesemer 1995 and earlier contributions from the 1960s cited there) sometimes called semi-endogenous. The steady state in the decreasing returns part and the upward sloping fertility part is a stable Malthusian one. On the very left we have a Malthusian one, which is unstable because of the increasing returns to scale. The second Smithian steady state in the falling part of the fertility curve is unstable because of the decreasing returns to scale. Now we derive some special cases for a given population growth curve:

Case 1: Productivity may be sufficiently low to let the Smithian steady states vanish.

Case 2: In this case there exists no decreasing-returns-to-scale part. This assumption is plausible for the rich countries and for the world (as in Foley's Loess fit regression), which are to some extent dominated by the rich countries, because the world as a whole has no signs of decreasing returns so far. Then the Malthusian, unstable steady state and the stable

Smithian one are still there. The other steady states are absent if the production function has no decreasing-returns-to-scale.

Case 3: The increasing returns part is weak and the production function has $x > 0$ at $N = 0$. If this value of x is high enough, the Malthusian unstable steady state vanishes. If, in addition, there is no decreasing-returns part, there is only one (Smithian) steady state.

Case 4: The production function has no increasing-returns-to-scale part. This assumption is a priori plausible for stagnant low-income and lower middle income countries. In this case the unstable, Smithian steady state and the stable Malthusian remain in the graph of figure 1. The two steady states more to the left vanish.

The two graphs in Figure 1 can be formalized as two linear-quadratic functions. Note that f-2 in Foley's model is not a percentage rate, but rather the change in population. Therefore we write the population growth curve as follows:

$$dN = a + by - cy^2 \qquad (1)$$

The production function is (according to the lower panel of Figure 1)

$$y = e + fN - gN^2 \qquad (2)$$

To both equations a time trend for historical shifts and a residual for econometric purposes can be added. Insertion of equation (2) into (1) yields a polynomial of the fourth degree:

$$dN = a + b(e + fN - gN^2) - c(e + fN - gN^2)^2 = \beta_0 + \beta_1 N + \beta_2 N^2 + \beta_3 N^3 + \beta_4 N^4 \qquad (3)$$

In principle such a polynomial can have four stationary values as Foley's model has. Estimation of the function would reveal whether or not this is the case. This follows from getting signs for coefficients and plotting the estimated function.

In case 1, parameters e and f must be low. In case 2, c is zero or at least very low such that the linear part is dominating. In case 3, e must be sufficiently large to guarantee that $x(N=0) > x(dN_{|f=2})$. In case 4, f would be zero.

Obviously, the special case of equation (3) of having only the linear part is Kremer's (1993) model and the linear-quadratic part is the well-known Verhulst-Pearl equation (see Gandolfo 1996). Foley's model extends of these cases by the terms with exponents three and four. The Verhulst-Pearl equation is of utmost importance here because of its well-known property to generate an unstable difference equation for parameters even if its continuous analogue is stable. This will turn out to be relevant for the forecasts below.

This is a very simple economic model. It has no physical or human capital or technical progress. In econometric terms we would therefore expect to have an omitted variable bias. The only justification for the simplicity we could think of then would be that it is producing good forecasts. These might indicate that the bias is small. Therefore we first focus on the forecasts.

Among the three classes of methods for the purpose of making population forecasts - forecasting time trends of level or growth rate of population, historical analogy, and social and economic theory (Lee 1991) – the use of equations like (3) or special cases of it is a combination of using economic theory and forecasting.

TABLE 1 OVER HERE

3 Population forecasting with Foley's model

We estimate equation (3) in the simple OLS mode. It is a differential equation and as such analysis of its stability is similar to a unit root test. We do not have the corresponding critical values for such analysis though.[1] In order to correct for autocorrelation we have added lagged dependent variables but they were mostly insignificant. In order to correct for endogeneity we have tried lagged right-hand side variables as instruments added to the other regressors in the GMM mode but the results for the forecasts are worse than those used below. Following Giersbergen and de Beer (1997) it is assumed that there is a lag of two years before population change, *dN*, reacts to changes in income, y. The results of our estimates are summarized in Table 1. In these estimates all exponential terms are significant at least at the 10% level and so is a squared time trend. However, when trying to run forecasts with these estimates it turns out that the difference equation is unstable, either because of the third (for high-income countries) or fourth exponent or because of the quadratic time trend. When dropping the terms causing instability, we get the regressions in Table 2. These estimates are still quite good in terms of significance, adjusted R-squared, absence of autocorrelation and heteroscedasticity and the Ramsey RESET test. The major disadvantage compared to the estimates in Table 1 is the increased number of observations indicating parameter non-constancy according to the one-step-forecast test and the n-step forecast test. The CUSUM of squares test is also worse. The forecasting results from these estimates are presented in the second but last line of Table 2 and in Figures A2-A6. The prediction for the world as a whole is 8.6 billion people. For high-income countries the prediction is 1.65 billion, for upper-middle income countries .78 billion, for lower-middle-income countries (including China) 2,76 billion and for lower income countries (including India) 5.3 or 4.3 for two different estimates (see Figures A6 and A7). These disaggregated predictions sum up to 10.5 or 9.5

[1] Similar models are used for the analysis of diffusion processes for new goods. The differential equations used there differ from those in the standard unit root tests. Unit roots go unmentioned (see Putsis and Srinivasan (2000). The justification for this may be that these models imply a different assumption on the true model generating the variable. For these models critical values are available only in exceptional cases (see Lucke and Lüthkepool 2004).

billion people. The forecast statistics, such as the Theil coefficient, indicate that the forecasts are reasonably good. In particular, the covariance proportion of the mean squared error is in all case above .99 with a correspondingly low bias proportion and variance proportion. The number of observations indicating doubts on parameter constancy is a weak point. However, rather than complicating the model or the estimation method, it may be better to repeat the forecast every year. Given its simplicity, this requires very little effort.

TABLE 2 OVER HERE

Our estimates are within the range of those obtained with other methods (see Table 3 in billion). This shows that Foley's model is useful for forecasts. In order to carry out the analysis in regard to the relevance of Malthus' ideas, we will plot next the graphs of the estimated equations and relate them to the special cases discussed above.

TABLE 3 OVER HERE

Figure A8 shows the plot of the regression used for population forecasting for the world in periods t = 0, 10, 100, 140 corresponding to the years 1960, 1970, 2060 and 2100. There are only two shifting or quasi steady states for given values of time. In spite of the time shift we will speak for simplicity of steady states. The one at a lower population is unstable and the other is stable. Over time the curve shifts upward. This might be due to technical progress, factor accumulation or other effects captured by the time trend. As a consequence the unstable steady state vanishes. Thus, here Foley is supported under the additional assumption of increasing returns: for the world as a whole history has no support for the ideas of Thomas Malthus, because the unstable steady state vanishes over time. The actual and forecasted population data of 3.06, 3.68, 8.03, 8.59 billion people for 1960, 1970, 2060 and 2100 indicate that the population is very close to the moving steady state. However, under either decreasing or increasing returns to scale the model has an unstable steady state to the left of the stable steady state. Without additional information we cannot know whether or not the stable one is the Smithian steady state.

The population growth curve for the high-income countries in Figure A9 looks very similar. The population numbers, H, for the years 1960, 1970, 2060 and 2100 are .6772, .758, 1.37, 1.65 billion. In later years these numbers are fairly close to the moving steady state, but not in earlier years. One can take it from the World Development Indicators that (not shown) the fertility rate for these countries has fallen below the value of two in the beginning of the seventies. Not being in the steady state is probably due to immigration as far as population is concerned and to technical progress in regard to GDP per capita. Under increasing (decreasing) returns to scale therefore it is plausible that the lower, vanishing steady state is the unstable (stable) Malthusian one, whereas the one that is approached is the (un-)stable Smithian one. However, again we can decide on this only when having more information in regard to scale economies.

For the upper-middle income countries we find only one steady state shown in Figure A10. Depending on the returns to population we may either have a stable but shifting Malthusian or Smithian one. The population, U, in 1960, 1970, 2060 and 2100 is .236, .294, .784, and .931 billion.

For lower middle-income countries we find two steady states again in Figure A11. The unstable steady state is vanishing as the curve moves up over time. Population growth is quite high. China is part of this group. The population, M, in 1960, 1870, 2060 and 2100 is 1.1, 1.35, 2.65, 2.76 billion, which is getting closer to the moving equilibrium.

For low-income countries there is only one moving steady state in Figure A12. The population, L, in 1960, 1870, 2060 and 2100 is 1.01, 1.27, 4.4, 5.31 billion. These values are very close to the steady state. The shifting stable steady state is either a Malthusian one or a Smithian one, again depending on the returns to scale.

So far this section has shown that Foley's model delivers reasonable forecasts and therefore may be useful in spite of the missing variables like physical and human capital. Next, we will estimate the model in its form with two equations in order to get the information on returns to scale. This will allow us to see whether or not Malthusian steady states and behaviour are irrelevant.

4 A system estimation of Foley's Model

In this section we estimate equations (1) and (2) as a system. As all variables have unit roots and we assume that they are cointegrated[2] we take first differences of the two equations and insert the lags of the level equations (1) and (2) for the lagged residual term and multiply it with a coefficient. This yields the following variant of an error-correction model.

$$d(dN)) = c(9)*[d(N(-1)) - c(1) - c(2)*y(-3) - c(3)*(y(-3))^2 - c(4)*t] + c(2)*d(y(-2))+c(3)*d(y(-2)^2)+c(4) +u(t)$$

$$d(y) = c(10)*[y(-1) - c(5) - c(6)*N(-1) - c(7)*(N(-1))^2 - c(8)*t] + c(6)*d(N)+c(7)*d(N)^2+c(8) +v(t)$$

The *c(i)* are the regression coefficients, and '*' indicates a multiplication. The added residuals are *u(t)* and *v(t)*. As all right-hand side variables are endogenous in the sense that a certain value of the residuals has an impact on the left-hand side variable and therefore on future values of the right-hand side variables, we use lagged regressors as instruments in a general method of moments (GMM) estimation. As the right-hand side has levels as well as first

[2] As pointed out in the previous section we do not have the critical values for unit root and cointegration tests that would correspond to the Foley model.

differences of some of the regressors we have to take care of interdependences among them in order to avoid linear dependence among instruments. We can use either the levels or the first differences as instruments. We tried both and the results were almost identical. The estimation results are summarized in Tables 4 and 5 and are plotted in Figures A13-A16.

TABLE 4 OVER HERE

For aggregate *world* data we find the result in column one of Table 4 and Figure A13. Time trends are highly insignificant and therefore have been dropped. As we will see below, the reason for this is that time trends of the population curve go into opposite directions in the groups with the different income levels and are positive for all productions functions but those of the low-income countries. The richer the countries according to the income classification the stronger the effect of the time trend (see Tables 4 and 5, coefficient c(8)). The population growth curve has the expected inverted u-shape. The production function has only the increasing returns part. By implication, there is an unstable Malthusian steady state and a stable Smithian one as Foley found it in his analysis. As the world is at higher population and income levels than the unstable steady state – in 2000 income per capita is 5640 constant 1995 US$ and the population is above 6 billion – population would continue to grow until income is about $9000 and population to about 10 billion. These values are at the higher end of the population prediction of the previous section. The value of $5640 is on the falling part of the population growth curve and therefore there is no Malthusian behaviour – increasing population change as income is higher - on the world level anymore.

In Figure A14 the result for the *high income* countries is plotted from the estimate of column two of Table 4 for the years 1960, 2000, 2060, 2100. The population change is increasing in income, but the time trend is pulling the curve down. This result is in accordance with recent literature arguing that it is not the income that reduces population growth but rather human capital formation and change in gender issues as captured by the time trend here. Children have positive income elasticity. The production function has a u-shaped form, but population is always to the left of the minimum of the curve: for the year 2000 we have N = .975 billion, x = 29200, dN > 0. By implication, we are in the Malthusian part of the fertility curve and the decreasing returns part of the production function. There are only Malthusian steady states. The relevant one to the left of the minimum of the production function is stable. However, the time trend shifts the population change curve down and the production function up. High-income countries therefore exhibit the classical constellation of time, or technical change and factor accumulation, versus decreasing returns and population growth, though under conditions of human capital formation. The time trend is strongly dominating the data. We feel that it is an open issue here whether or not the upward sloping part of the population growth curve has to be interpreted in the Malthusian spirit. It is equally plausible to argue that children are like a luxury good. This then should not be interpreted as Malthusian behaviour, in particular because the phenomenon does not appear in the estimated model for the other income groups.

For *upper middle income* countries the results are summarized in column 3 of Table 4 and Figure A15. The production curve has decreasing returns to scale and the population curve has an inverted u-shape. A time trend shifts the population curve down and the production function upward. By implication, we have a shifting stable Malthusian steady state and an unstable Smithian one. For the year 2000 these countries are near the peak of the population curve with y = 4750 and N = .5 billion. Malthusian behaviour therefore is essentially absent. The shifting time trend is stronger than the forces of stability and again dominates the process ensuring the growth of this group.

TABLE 5 OVER HERE

For *lower middle income* countries the results are summarized in the first column of Table 5 and in Figure A16 for the years 1960 and 2000 only. Both curves are falling in the relevant range as we have y = 500 and 1400, and N = 1.01 and 2.5 billion in 1960 and 2000 respectively. Therefore Malthusian behaviour is absent. The time trend shifts both curves up. The time trend therefore works in favour of per capita income growth and population growth works against it. As the data show the positive impact of the time trend on income is larger than that of population growth and these countries are growing in per capita terms. Part of this though is due to growth in China, which is included in this group. There is no steady state, not even in the shifting form.

The *low-income* countries differ from the lower middle-income group in that the time trend has no impact on the production function. The results are shown in the last column of Table 5 and in Figure A17. The production function does not shift and therefore is plotted only once. Both curves have inverted u-shape and therefore Figure A17 is the one that resembles Foley's model most closely. The population growth curve for 1960 allows for constant populations at incomes of about \$100 and \$650 and in 2000 for \$40 and \$740. The income levels for 1960 exist on the production function and therefore all four steady states of Foley's model as of Figure 1 exist. The maximum income of the production function is \$735.85. By implication the Smithian steady states vanish through the shift in the population growth curve around the year 2000. In the year 2060 the Malthusian steady states have also vanished. Actually the prospects for the low-income countries are worse than Malthus thought. The population growth makes even a Malthusian steady state vanish. In the year 2001 the income in this sample is at \$480. This is in the falling part of the population growth curve. Therefore we again cannot find Malthusian behaviour. When population reaches a value of 6.5 billion, the income per capita reaches a maximum and thereafter declines. This value of the population is higher than the one we predicted for 2100 in the previous section and therefore will be reached only after 2100. This is a long time to go and therefore there is a big chance that population growth may induce a structural break of our estimates, which may bring about technical change or stop the shift in the population growth curve. This caveat is an optimistic one. The pessimistic ones are that either no structural break comes about and population remains growing or the growth comes from India, which is in this country set. Therefore we go to search for a different classification of countries in the next section. Before doing so we present some results for Sub-Saharan Africa.

For Sub-Saharan Africa the results are summarized in the last column of Table 5 and Figure A18. The figure resembles that of lower middle-income countries.[3] There is no steady state and there is positive population growth with an upward shifting curve and there are decreasing returns in an upward shifting production function. Again the question is whether or not the time trend in the production function is able to outweigh the impact of population growth driving the economies into decreasing returns and decreasing per capita income. The data though are much less favourable than for lower middle-income countries. The Sub-Sahara region has had positive growth rate for 1960-1974, then negative rates until 1994 and again positive ones in the most recent period. The average growth rate for the whole period is an insignificant one of .1%. By implication, the problem of population growth outweighing the time trend is clearly present in Sub-Saharan Africa although there is no steady state and the population change behaves essentially in a Smithian way as income decreases population growth.

[3] Most countries are also in the low-income group. However, there are four countries with an income above \$4000: Seychelles, South-Africa, Botswana, Mauritius and Gabon. They therefore belong to the group of upper-middle income countries (\$2936-9075). In the lower-middle income group (\$735-2935) are Cote d'Ívoire, Namibia, Swaziland.

For the larger sets of countries the question now is whether or not the results for lower middle income countries are based on the performance of China and for low income countries on those for India and those for SSA embellished by their richer countries (see footnote above). In the classifications used so far we cannot reject Foley's claim that history has been unfriendly to the ideas of Malthus in regard to Malthusian behaviour. But the problem of population pressure is still present for Sub-Saharan Africa but not for the larger groups including India and China. This can either be a consequence of the data or of the model. In this paper we keep sticking to the Foley's model and continue investigating the data.

TABLE 6 OVER HERE

5 The empirical income stagnation of low and some lower middle-income countries

For poor countries there is little doubt that population growth slows down GDP per capita growth whereas other factors enhance it. The crucial question in terms of finding the net effect therefore is to see how large the growth was and whether or not it was about zero for some countries. We plot the Kernel density estimates for GDP per capita first for a set of countries for which we have data since 1960 in A19 and in Figure A20 for all countries with data available in the respective years. The first approach follows a constant set of countries over time as suggested by Quah (1996) and the second looks at the updated distribution in each time period. The restricted data set confirms Quah's twin peak result. In the unrestricted data set the second peak is somewhat diluted. Both Figures show that there is always a peak at around 1000 constant 1995 dollars. It is hard to detect growth of the big peak with the mere eye. The local minimum following the big peak moves to a value of $10000 during the decennia. By introspection we form the guess that there is little growth for countries below $1200. Moreover, van den Berg (2001) suggests that per capita income at the end of the Malthusian phase for the world at 1820 of at $651 according to Maddison (1995) or $250 from Pritchett (1997). In Table 6, in the left part, we present counts telling how many countries had per capita incomes below these three values of $250, $651, and $1200 in 1995 dollars[4]. As the number of countries for which data are available is increasing too, we take these numbers as percentage of all countries for which data are available. The percentage of countries, which is below the two Malthusian values of $250 and $651, has not improved during the period 1975-2000. For our value of $1200 the percentage being above it has not improved since 1980. By and large the two descriptive methods of Figures A19 and A20 and Table 6 indicate that there may be a no-growth club of countries with incomes below $1200. For the year 2002 there are 71 countries with per capita income below $1200 (henceforth U1200). As India and China are among them we do not aggregate the data but rather consider a panel of countries, which give less weight to them.

TABLE 7 OVER HERE

In Table 7 we present the results from the regression of the log of the GDP per capita of the U1200 countries on a country-specific constant and a common time trend. The coefficient for the time trend is well known to be the average growth rate. It is positive for the whole period 1960-2002 but as small as .0034, which is less than a half percent. It is a half percent during the last dozen of years 1991-2002. For all other periods it is negative. In particular, for the period 1970-2002 the result from Table 6 is confirmed saying that there is negative growth for a period of 33 years, including the two oil crises, the lost decade following the 1981/82 debt crisis, and also including the period from 1991-2002 with positive

[4] However, in Pritchett's paper these are 1985 purchasing power equivalents, corresponding to $70 in 1985 dollars. Even when transforming them into 1995 dollars there are only very few countries in that range. For our purpose of finding a set of countries with almost no growth we need a higher level. With about 9% of countries with data availability under $250 in 1995 dollars we do not want to go to lower levels with lower percentages. Maddison's data were in 1990 dollars, which is a small difference and there is no need here to meet Maddison's number exactly.

growth. This latter period does not compensate for the negative growth of the two previous decennia.

TABLE 8 OVER HERE

The results of a panel data estimate for the u1200 group are summarized in Table 8. It is a fixed effects estimate. The population change equation and the production function are estimated separately because the interdependence of the equation in the estimates for aggregated data was very weak when comparing least squares and SUR (seemingly unrelated regression) estimates, both not shown. Moreover, we do not use a common factor restriction and we estimate in the SUR method because adding instruments in GMM leads to a ‘near singular matrix warning’ and the SUR method makes sure that the residuals of the countries are not taken as independent but rather the population growth (and the production function, each separately) equations are allowed to have correlated error terms. For the population growth equation this is plausible in particular because of the aids epidemic, and for the production function because of the interrelated business cycles and growth processes. Moreover, in the estimation of the production function we can improve the estimate considerably by allowing for country-specific time trends. When having the estimation results we extract the coefficient of population and GDP per capita respectively as a common factor from the level variables, set the level term equal to zero and get the following equation in terms of the coefficients of Table 8:

$$dN=\frac{798000}{0.655752}+\frac{-836.7945}{0.655752}y+\frac{0.357587}{0.655752}y^2+\frac{3688.020}{0.655752}\times t$$

$$x=\frac{37.62}{0.083796}-\frac{7.05\times10^{-07}}{0.083796}N+\frac{3.39\times10^{-16}}{0.083796}N^2+\frac{.626}{0.083796}t$$

In these equations and in the plot in Figure A21 we use the average values of the fixed effects in both equations and of the time trends of the second equation. In Table 8 we have given the intervals in which they are in the estimates. Figure A21 shows that population behaviour in the u1200 group is essentially Smithian and there are decreasing returns to scale. Therefore Malthus' idea does not come out here but the population problem is of course keeping growth down. The panel average of per capita income is y = \$475 and N = 38mln, and for the years 2000-2002 they are y = \$494 and N=52.5mln. The latter average number indicates that the 71 countries account for 3.5 billion people, which is more than half of the world population in the no growth club. However, once India and China are taken out the club size is 1 billion people. On the other hand then the income limit of \$1200 might be larger as well. Again, an income change from \$475 to \$494 within twenty years (from the middle of the panel in 1981 to 2001 for the restricted panel with years 2000-2001, this is one dollar a year and 1/5th of a percent) confirms that there is little growth in GDP per capita but a high one in population growth. The prospects are of course worst for countries with negative time trends. The distribution of the coefficients of the time trend variable is plotted in A22. The countries with the negative time trends are: Bolivia, Burundi, Central African Republic, Comoros, Congo DR, Cote d'Ivoire, Eritrea, Gambia, Georgia, Guinea-Bissau, Haiti, Kiribati, Kyrgyz Rep., Liberia, Madagascar, Mauritania, Moldova, Mongolia, Nicaragua, Niger, Papua New Guinea, Sierra Leone, Solomon Island, Tajikistan, Togo, Vanuatu, West Bank and Gaza, Zambia, Zimbabwe. This is roughly what you would expect and again confirms that Foley's model is not only simple but also very realistic.

6. Summary and conclusion

In section three, we have shown that Foley's model produces reasonable forecasts for the population growth of the aggregated country groups according to the Worldbank income classification. In section four, we found plausible estimates of the two-equation system version of Foley's model. Malthusian behaviour is hard to detect unless one accepts that it is present in high-income countries whereas it is clearly absent in all other groups of countries. An interpretation of children having a high-income elasticity, or, richer parents can afford to have more children, seems more plausible. Foley's increasing returns result is found on the world level and for low-income countries. For the other group we find decreasing returns to population growth. As aggregate data include India in the low income group and China in the lower-middle income group one gets even the impression here that a problem of population growth is present only in Sub-Saharan Africa. Therefore we continue with even more disaggregated data. In section three we show that (i) the density distribution of world income has a stagnant peak around \$1000 and (ii) that the percentage of countries that is below the income levels of \$250 and \$650 has not improved since 1975 and (iii) for the level of \$1200 there is stagnation since 1980. The suspicion that the u1200 group is a no growth club – with the exceptions of individual countries of course – is confirmed by panel regression analysis. For the period 1975-2002 the panel average growth rate is negative in spite of the fact that it is positive for the years 1991-2002. The estimation of Foley's model for the u1200 group shows a curvature that resembles that of the lower middle-income countries and Sub-Saharan Africa. However, the curves show lower population change and lower incomes because the large and rich countries get a higher weight in a panel compared to aggregated data. In the u1200 group Malthusian behaviour is absent, but population growth outweighs the growth effects of a positive time trend under conditions of decreasing returns to scale. This seems to be closer to Ricardo's version of the population growth problem than to that of Malthus.

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Table 1 Least squares estimation and test of Foley's Model

Dependent variable: Population change

Variable	**Group**	**World**	**HI**	**UMI**	**LMI**	**LI**
constant		-4.26E+09	4.94E+09	1.29E+08	-3.19E+09	-5.93E+08
		(-9.757049)	-(1.771745)	-(7.412103)	-(10.591800)	-(4.615645)
Pop(-2)		3.934045	-25.69691	-1.507849	8.312608	1.647432
		(-9.839606)	(-1.814914)	(-7.361738)	(-10.63889)	(-4.656245)
$Pop(-2)^2$		-1.28E-09	4.98E-08	6.68E-09	-7.86E-09	-1.45E-09
		(-9.769586)	(-1.858241)	(-7.540755)	(-10.57153)	(-4.552366)
$Pop(-2)^3$		1.78E-19	-4.23E-17	-1.27E-17	3.25E-18	4.89E-19
		(-9.682538)	(-1.893228)	(-7.580521)	-(10.508680)	(-4.498179)
$Pop(-2)^4$		-9.77E-30	1.32E-26	8.73E-27	-5.15E-28	-6.68E-29
		(-9.695672)	(-1.911164)	(-7.507024)	(-10.56836)	(-4.506314)
Time		-	-	-		
		-	-	-		
$time^2$		565861.7	33894.73	-	172386.6	300334
		-(7.897)	-(3.898)	-	-(7.583)	-(4.500)
$adjR^2$		0.917279	0.729537	0.938222	0.839557	0.990539
JarqBera F-Prob		0.30501	0.225185	0.698856	0	0.011939
Breusch-Godf F-Prob		0.05774 (1)	0.0876 (1)	0.0404 (1)	0.840391	0.003307
White hetsc. F-Prob.		0.033589	0.796366	0.496954	0	0.030769
Ram. Res. T. F-Prob		0.002644(2)	0.13 (2)	0.006973	0.129586(1)	0.060027(1)
Cusum		5%	5%	5%	5%	5%
Cusum squared		>5%(1990)	>5%	>5%(96-99)	5%	>>5%
No. Osf		4,<10%	5,<15%	4, <15%	4.<15%	11,<15%
No. nsf		6, <15%	10<10%	15, <15%	4,<15%	30, <15%

Legend:

t-values according to the Newey-West HAC in parantheses

Cusum (sq) cumulated sum of (squared) residuals

No. osf Number of observations of one step forecast test doubting parameter constancy

No. nsf Number of observations of n step forecast test doubting parameter constancy

Table 2 Least squares estimation for forecasting with Foley's Model

Dependent variable: Population change

Variable	**Group**	**World**	**HI**	**UMI**	**LMI**	**LI (1)**	**LI (2)**
constant		-4.46E+08	-1.02E+08	1.99E+08	-5.37E+08	7.63E+08	3.95E+08
		-(1.218)	-(1.583)	-(5.986)	-(1.401)	(5.547)	(2.642)
Pop(-2)		0.926109	0.355784	-1.347578	1.661487	-1.013044	-0.566764
		-(2.980)	-(1.879)	-(5.907)	-(1.625)	-(5.397)	-(2.561)
$Pop(-2)^2$		-4.31E-10	-2.84E-10	2.68E-09	-1.75E-09	2.90E-10	2.12E-10
		-(4.189)	-(2.037)	-(6.072)	-(1.769)	(5.154)	(2.758)
$Pop(-2)^3$		6.99E-20-		-2.44E-18	7.65E-19	-5.37E-20	-3.58E-20
		-(4.534)-		-(6.282)	-(1.852)	-(5.578)	-(2.971)
$Pop(-2)^4$		-4.11E-30-		-	-1.24E-28-		-
		-(4.806)-		-	-(1.939)-		-
Time		16900021-		2668011	2861633	13645096	6755729
		-(6.200)-		(-5.87539)	-(2.671)	(5.667)	(2.661)
time sq		-	14561.14-		-	125761.1	
		-	-(2.640)-		-	(3.923)	
d(po(-1))		-	-	-	0.406567-		
					-(3.771)-		
adjR ^{2}		0.889969	0.689251	0.889939	0.879317	0.99237	0.986626
JarqBera F-Prob		0.302285	0.478301	0.785088	0.161876	0.000581	0
Breusch-Godf F-Prob		0.0000440.0112 (1)		0.0003940.098986 (1)		0.012136 (1)	0.001816(5)
White hetsc. F-Prob.		0.167455	0.110201	0.000001	0.000001	0.087351	0.001636
Ram. Res. T. F-Prob (Nft)		0 (2)	0.0683 (1)	7.4E-05 (1)	0.01453 (2)	0.109457 (4)	0.000252(2)
Cusum		5% (1993)	5%	5%	5%	5%	5%
Cusum squared		>5%	5%>5% (81-99)		>5%	>5%(75-92)	>5% (1975-92)
No. Osf		9, <15%	3, <10%	4, <5%	11, <15%	11, <10%	11, <10%
No. nsf		23, bef.1995	2, 5%	29, <15%	17, bef.1988	25, <1%	all,10%
Pop 2100		8594022559	1653189186	779214678	2.76E+09	5.31E+09	4.29E+09
Sum Pop 2100		1.0502E+10with Li(1), or		9.48E+09with Li(2)			

Legend:

t-values according to the Newey_West HAC in parantheses

Cusum (sq) cumulated sum of (squared) residuals

No. osf Number of observations of one step forecast test doubting parameter constancy

No. nsf Number of observations of n step forecast test doubting parameter constancy

Table 3

Recent forecasts	**period**	**Forecasts from the 1980s for 2100**	
UN 5.5-14	**2100**	**UN 1983**	**7-14**
IIASA 9	**2070**	**UN 1988**	**10.8**
Foley 7.5-8.5	**2100**	**Worldbank 1988**	**10.5**

This paper:
For aggregate world data 8.6;
by Worldbank income groups 10.5 or 9.5

Sources: Foley (2000) using Loess fits; Documents (2003); Lee (1991); this paper

Table 4 Results of System Regression for High and Middle Income Countries

Coefficient (t-value)	Variable\Group	World [a]	high inc. [b]	upp.mid.inc. [c]
Dependent variable: (First difference of) population change				
c(1)	constant	-8848829.00	6335751.00	-10132.10
		-(.33)	(6.18)	-(.01)
c(2)	GDPpc(-2)	38782.64	237.60	3803.16
		(3.87)	(2.06)	(7.32)
c(3)	(GDPpc(-2))^2	-4.22	0.02	-0.43
		-(4.44)	(11.62)	-(5.53)
c(4)	time	-	-508827.70	-49755.87
			-(9.90)	-(4.52)
c(9)	error corr.term	-0.10	-0.81	-0.18
		-(2.53)	-(17.58)	-(2.02)
Dependent variable: (Change of) GDP pc				
c(5)	constant	1048.29	98026.44	12555.69
		(1.68)	(2.73)	(4.27)
c(6)	population	0.00	0.00	0.00
		(2.78)	-(2.13)	-(3.24)
c(7)	Population ^{2}	0.00	0.00	0.00
		(.21)	(1.31)	-(2.26)
c(8)	time	-	910.15	768.19
			(8.45)	(3.95)
c(10)	error corr.term	-0.25	-0.16	-0.15
		-(6.31)	-(4.74)	-(2.51)
Adj. R^2		0.11/.16	0.335/.075	0.07/.25
DW		1.40/1.50	1.93/1.39	2.08/1.58
J-statistic		0.254	0.281	0.205
no.obs. = n		37/36	36/38	38/40
nJ (1st eq.)<chi-sq.(d.f.),5%lev. [d]		37x.25<16(9)	36x.28<21(12)	38x.2<11(5)
nJ (2nd eq.)<chi-sq.(d.f.),5%lev.		36x.25<18(10)	38x.28<21(12)	40x.2<11(5)

(a) Instruments: 1st eq.: c d(N(-1)) y(-3) y(-3)^2 d(y(-2)) d(y(-2)^2) d(N(-2)) d(y(-3)) d(y(-3)^2) d(N(-3)) d(y(-4)) d(y(-4)^2) d(N(-4))
2nd eq.: c y(-1) N(-1) N(-1)^2 d(N) d(N^2) d(y(-3)) d(N(-2)) d(N(-2)^2) d(y(-4)) d(N(-3)) d(N(-3)^2) d(y(-5)) d(N(-4))
(b) Instruments: 1st eq.: c d(N(-1)) y(-3) y(-3)^2 t d(y(-2)) d(y(-2)^2) d(N(-2)) d(N(-3)) d(N(-4)) d(y(-3)) d(y(-3)^2) d(y(-4)) d(y(-4)^2) d(y(-5)) d(y(-5) ^{2})
2nd eq.: c y(-1) N(-1) N(-1)^2 t d(N) d(N^2) y(-2) y(-3) y(-4) d(N(-1)) d((N(-1))^2) d(N(-2)) d((N(-2))^2) d(N(-3)) d((N(-3))^2)
(c) Instruments: 1st eq.: c d(N(-1)) y(-3) y(-3)^2 t d(y(-2)) d(y(-2)^2) d(N(-2)) d(y(-3)) d(y(-3)^2);
2nd eq.: c y(-1) N(-1) N(-1)^2 t d(N) d(N^2) y(-2) d(N(-1)) d((N(-1))^2)
(d) The number for the degrees of freedom is equal to the number of constraints in the equation plus the number of instruments going beyond the number of regressors.

Table 5

Results of System Regression for lower middle and low income countries and Sub-Saharan Africa

Dependent variable: (First difference of) population change

Coefficient (t-value)	Variable\Group	low.mid.inc.[d]	lower inc. [e]	SSA [f]
c(1)	constant	153000000.00	-23788013.00	9475210.0
		(1.85)	-(3.48)	(1.98)
c(2)	GDPpc(-2)	-274399.90	281118.30	-18839.6
		-(5.91)	(7.62)	-(1.21)
c(3)	$(GDPpc(-2))^2$	140.18	-380.30	18.9
		(6.17)	-(11.22)	(1.50)
c(4)	time	4815680.00	452914.40	267950.2
		(1.57)	(4.97)	(48.37)
c(9)	error corr.term	0.04	-0.35	-0.5
		(1.68)	-(8.04)	-(19.05)
Dependent variable: (Change of) GDP pc				
c(5)	constant	1898.40	30.54	2623.2
		(2.29)	(1.82)	(46.71)
c(6)	population	0.00	0.00	0.0
		-(1.69)	(11.94)	-(34.89)
c(7)	$population^2$	0.00	0.00	0.0
		-(.48)	-(3.42)	(33.48)
c(8)	time	68.01	-	80.7
		(4.60)		(33.65)
c(10)	error corr.term	-0.18	-0.41	-0.5
		-(3.00)	-(7.49)	-(23.04)
Adj. R^2		.08/.096	.095/.14	.12/.31
DW		1.26/1.09	1.4/1.35	1.67/1.40
J-statistic		0.247	0.224	0.278
no.obs. = n		33/35	37/38	35/37
nJ (1st eq.)<chi-sq.(d.f.),5%lev [g]		33x.25<12.5(6)	37x.224<16.9(9)	35x.28<28(18)
nJ (2nd eq.)<chi-sq.(d.f.),5%lev.		35x.25<12.5(6)	38x.224<19.6(11)	37x.28<28(18)

(d) Instruments: 1^{st} eq.: c d(N(-1)) y(-3) $y(-3)^2$ t d(y(-2)) $d(y(-2)^2)$ d(N(-2)) d(y(-3)) $d(y(-3)^2)$;
2nd eq.: c y(-1) N(-1) $N(-1)^2$ t d(N) $d(N^2)$ y(-2) d(N(-1)) $d((N(-1))^2)$
(e) Instruments: 1^{st} eq.: c d(N(-1)) y(-3) $y(-3)^2$ t d(y(-2)) $d(y(-2)^2)$ d(N(-2)) y(-4) $y(-4)^2$ d(N(-3)) y(-5) $y(-5)^2$;
2^{nd} eq. : c y(-1) N(-1) $N(-1)^2$ d(N) $d(N^2)$ y(-2) N(-2) $N(-2)^2$ y(-3) N(-3) $N(-3)^2$ y(-4) N(-4) $N(-4)^2$
(f) Instruments: 1^{st} eq: c d(ssa(-1)) yssa(-3) yssa(-3)^2 t d(yssa(-2)) d(yssa(-2)^2) d(ssa(-2)) d(yssa(-3)) d(yssa(-3)^2) d(ssa(-3)) d(yssa(-4)) d(yssa(-4)^2) d(ssa(-4)) d(yssa(-5)) d(yssa(-5)^2) d(ssa(-5)) d(yssa(-6)) d(yssa(-6)^2) d(ssa(-6)) d(yssa(-7)) d(yssa(-7)^2)
2nd eq.: c yssa(-1) ssa(-1) ssa(-1)^2 t d(ssa) d(ssa^2) yssa(-2) d(ssa(-1)) d(ssa(-1)^2) yssa(-3) d(ssa(-2)) d(ssa(-2)^2) yssa(-4) d(ssa(-3)) d(ssa(-3)^2) yssa(-5) d(ssa(-4)) d(ssa(-4)^2) yssa(-6) d(ssa(-5)) d(ssa(-5)^2)
(g) The number for the degrees of freedom is equal to the number of constraints in the equation plus the number of instruments going beyond the number of regressors.

Table 6

Number of countries with GDP per capita (constant 1995$) not higher than					Percentage of countries with GDP per capita [a]			
year	$250	$651	$1200	obs.available	year	< $250	< $651	< $1200
1960	14	39	55	101	1960	0.138614	0.386138614	0.544554
1970	12	40	59	118	1970	0.101695	0.338983051	0.5
1973	11	37	54	120	1973	0.091667	0.308333333	0.45
1975	11	35	54	123	1975	0.089431	0.284552846	0.439024
1980	12	37	55	140	1980	**0.085714**	**0.264285714**	**0.392857**
1990	15	50	69	174	1990	**0.086207**	**0.287356322**	**0.396552**
2000	16	52	70	178	2000	**0.089888**	**0.292134831**	**0.393258**

Source:author's counts based on WDI 2003

(a) Percentage of observations available.

Table 7	**Average growth rate of countries with GDP per capita below $1200[c]**				
Period	1960-2002	1965-2002	1970-2002	1980-2002	1991-2002
growth rate [a]	0.0034	0.00156	-0.0007	-0.0024	0.005
(t-value) [b]	4.98	2.2	-1.09	-3.25	1.96
adj.R-square	0.77	0.786	0.8	0.82	0.94
Countries	71	71	71	71	71
periods	43	38	33	28	12
total obs.	2342	2147	1937	1712	845

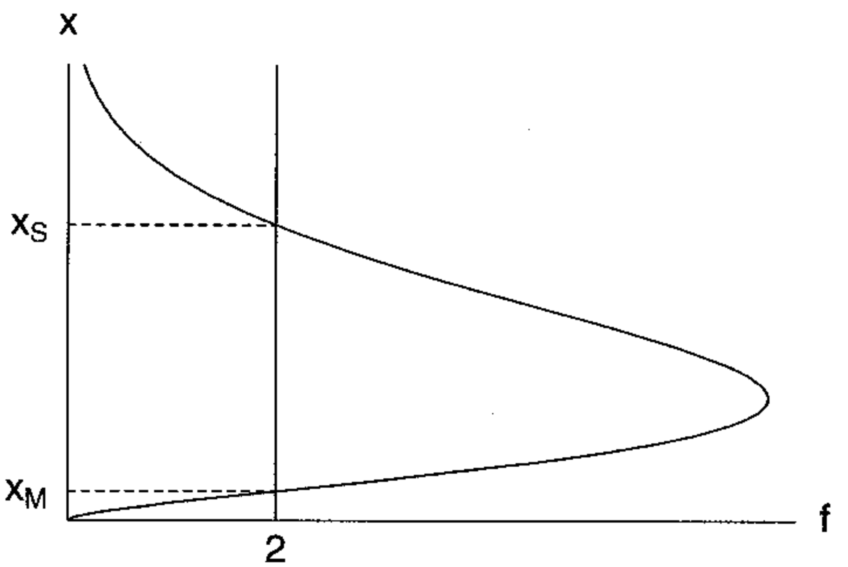
(a) The estimate is done with the fixed effect mode using country specific intercepts but a common coefficient for the time trend.
(b) t-values are based on panel corrected standard errors (PCSE) and the seemingly unrelated regression method (SUR).
(c) 1995 dollars

Table 8

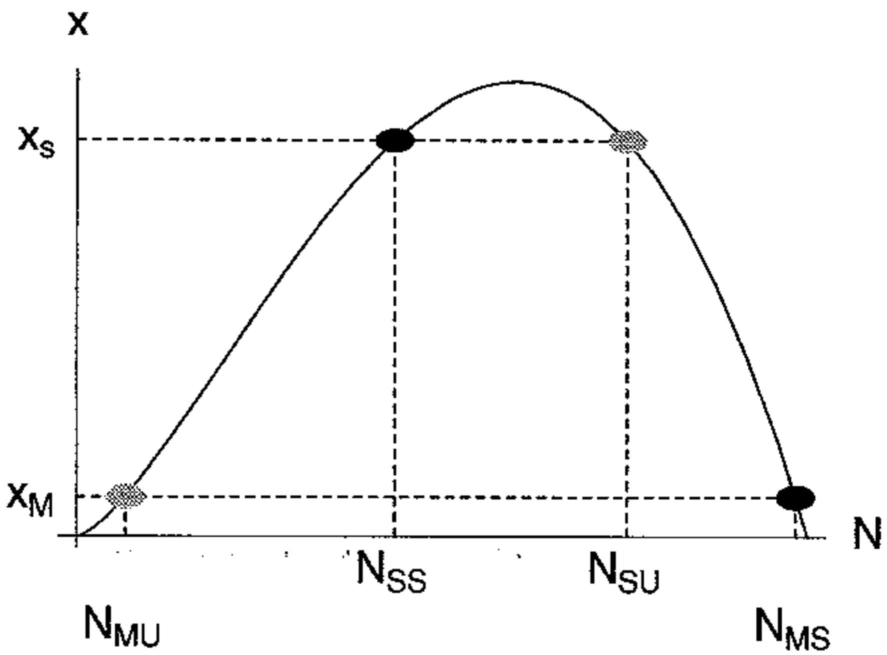
Results of Panel Regressions for countries with GDP per capita below $1200

Depend. variable:	Change of Population change		Change of GDP per capita
	Coefficient	Coefficient	
Variable	(t-value)	(t-value)	
constant	(52e3,10e6)	constant	(-863,1272)
	all > 3		
d(pop(-1))	-0.66	GDPpc(-1)	-0.083796
	-(65.03)		-(29.08)
GDPpc(-3)	-836.79	pop(-1)	-7.05E-07
	-(66.70)		-(31.68)
$(GDPpc(-3))^2$	0.358	pop(-1)^2	3.39E-16
	(66.07)		(33.34)
time	3688.02	time	(-32,24)
	(65.99)		
d(GDPpc(-2))	-1090.212	d(pop)	3.16E-06
	-(161.59)		(21.54)
$d((GDPpc(-2))^2)$	0.419	d(pop^2)	-1.66E-15
	(113.82)		-(20.30)
Adj. R^2	0.32		0.13
DW	2.33		1.27
no.obs. = n	2129 (40x71)		2171 (42x71)

Figure 1



The theoretical income–fertility relation has both an upward sloping segment, corresponding to Malthus' assumption of rising fertility with income, and a downward sloping segment, corresponding to the demographic transition in which fertility falls with income. There are two equilibrium levels of per-capita output, at which total fertility equals 2, the Malthusian equilibrium x_M , and the Smithian equilibrium, x .

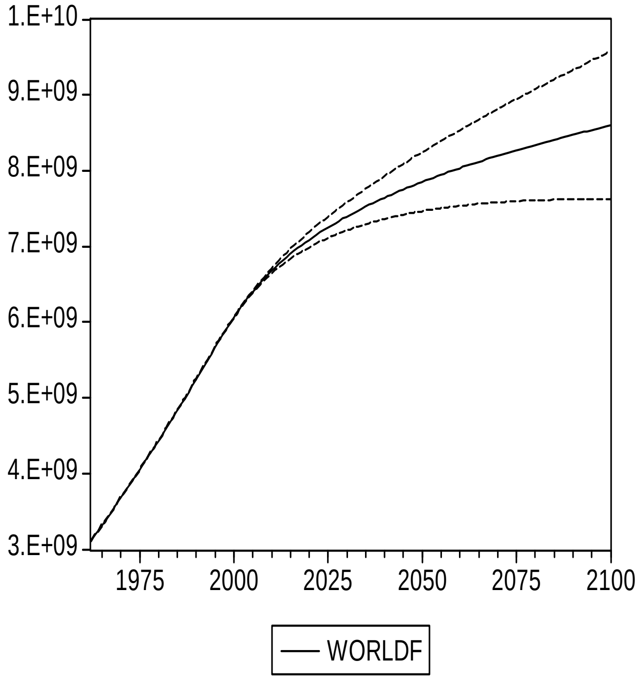


When the population–per-capita output relation has both a rising portion, representing the effects of the division of labor, and a falling portion, representing the effects of diminishing returns, there are potentially two Malthusian and two Smithian equilibria. The low-population Malthusian equilibrium and the high-population Smithian equilibrium are unstable, while the low-population Smithian equilibrium and the high population Malthusian equilibrium are stable.

Source: Foley 2000

Appendix: Figures

Figure A2 World Population forecast



Forecast: WORLDF
Actual: WORLD
Forecast sample: 1960 2100
Adjusted sample: 1962 2100
Included observations: 40

Root Mean Squared Error	4841278.
Mean Absolute Error	3939963.
Mean Abs. Percent Error	0.087420
Theil Inequality Coefficient	0.000518
Bias Proportion	0.000265
Variance Proportion	0.000504
Covariance Proportion	0.999231

Figure A3 Population forecast high income countries

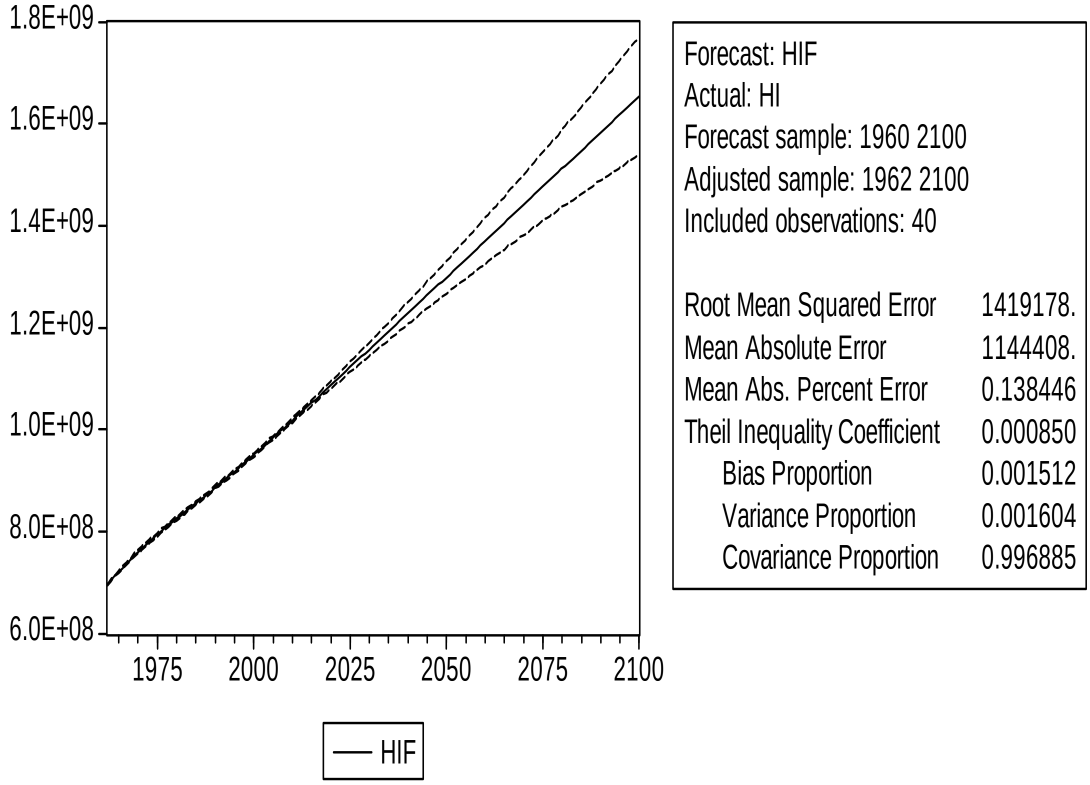
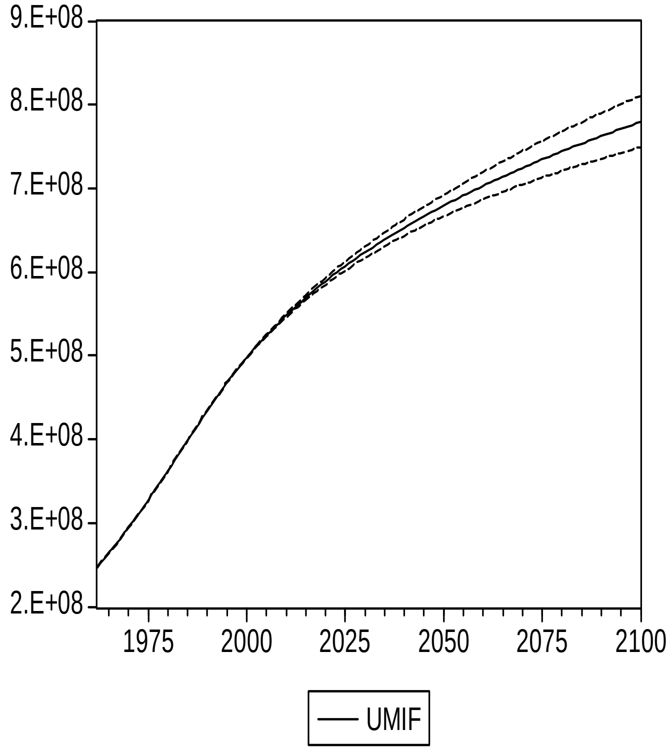


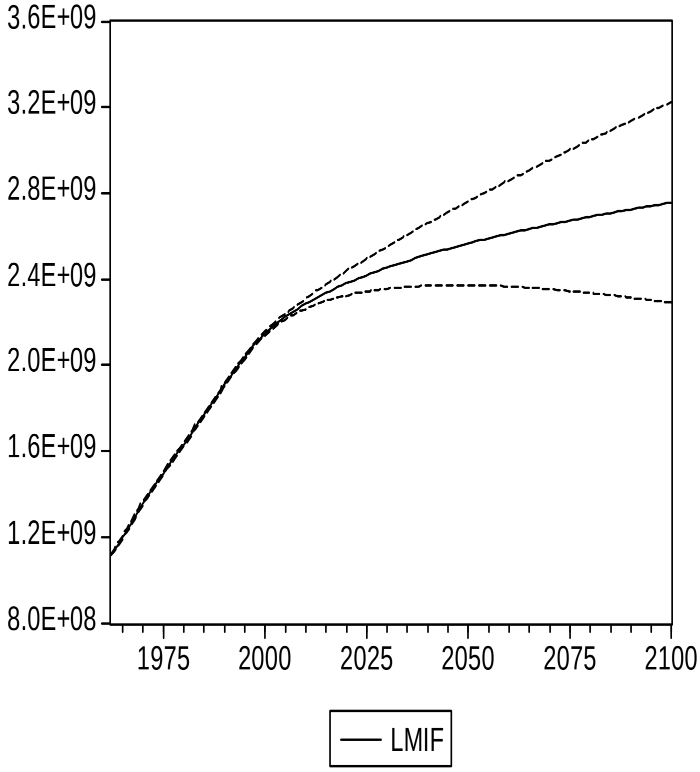
Figure A4 Population forecast Upper Middle Income Countries



Forecast: UMIF
Actual: UMI
Forecast sample: 1960 2100
Adjusted sample: 1962 2100
Included observations: 40

Root Mean Squared Error	344217.7
Mean Absolute Error	279121.0
Mean Abs. Percent Error	0.075565
Theil Inequality Coefficient	0.000450
Bias Proportion	0.002988
Variance Proportion	0.006741
Covariance Proportion	0.990271

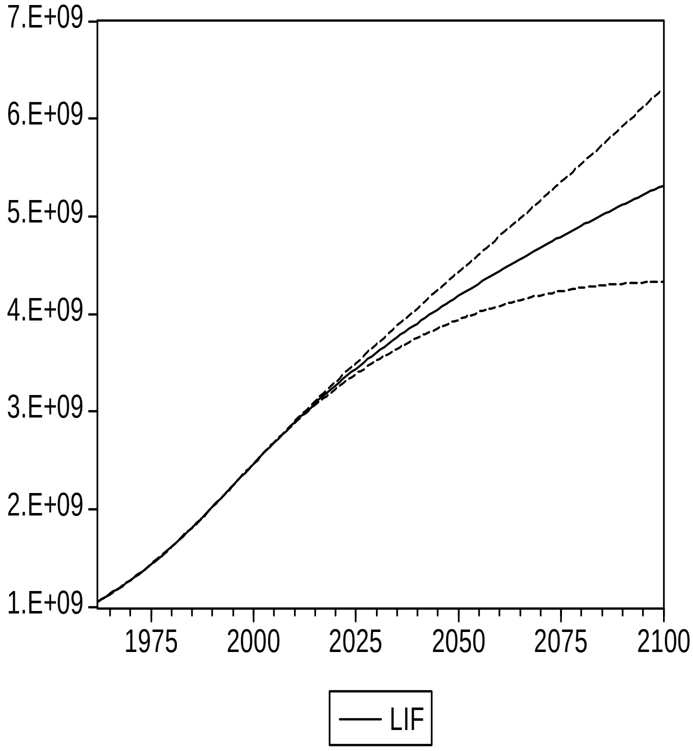
Figure A5 Population Forecast Lower Middle Income Countries



Forecast: LMIF
Actual: LMI
Forecast sample: 1960 2100
Adjusted sample: 1962 2100
Included observations: 40

Root Mean Squared Error	3782967.
Mean Absolute Error	3194104.
Mean Abs. Percent Error	0.198864
Theil Inequality Coefficient	0.001116
Bias Proportion	0.000542
Variance Proportion	0.001474
Covariance Proportion	0.997984

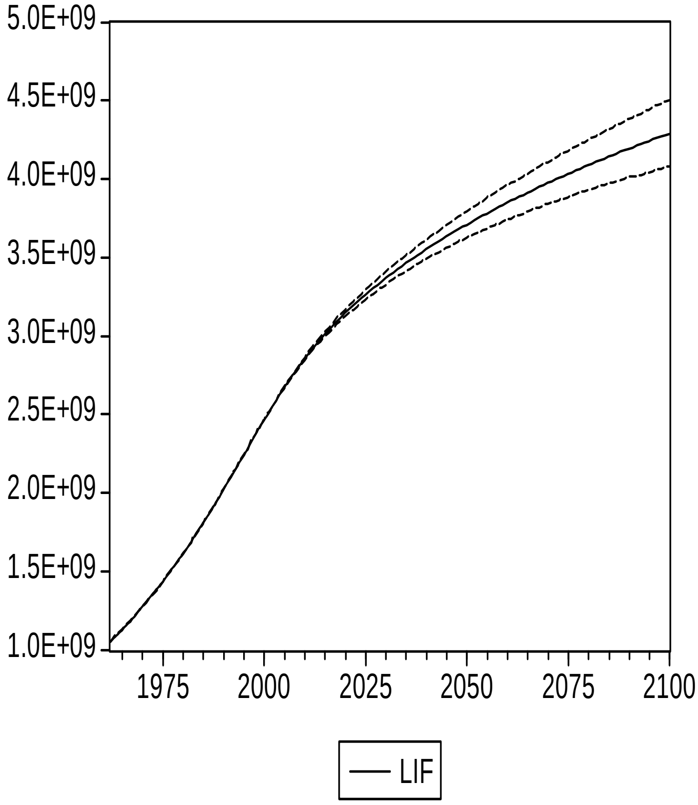
Figure A6 Population Forecast Lower Income Countries, (1)



Forecast: LIF
Actual: LI
Forecast sample: 1960 2100
Adjusted sample: 1962 2100
Included observations: 40

Root Mean Squared Error	1172513.
Mean Absolute Error	815149.9
Mean Abs. Percent Error	0.045442
Theil Inequality Coefficient	0.000332
Bias Proportion	0.000025
Variance Proportion	0.000115
Covariance Proportion	0.999860

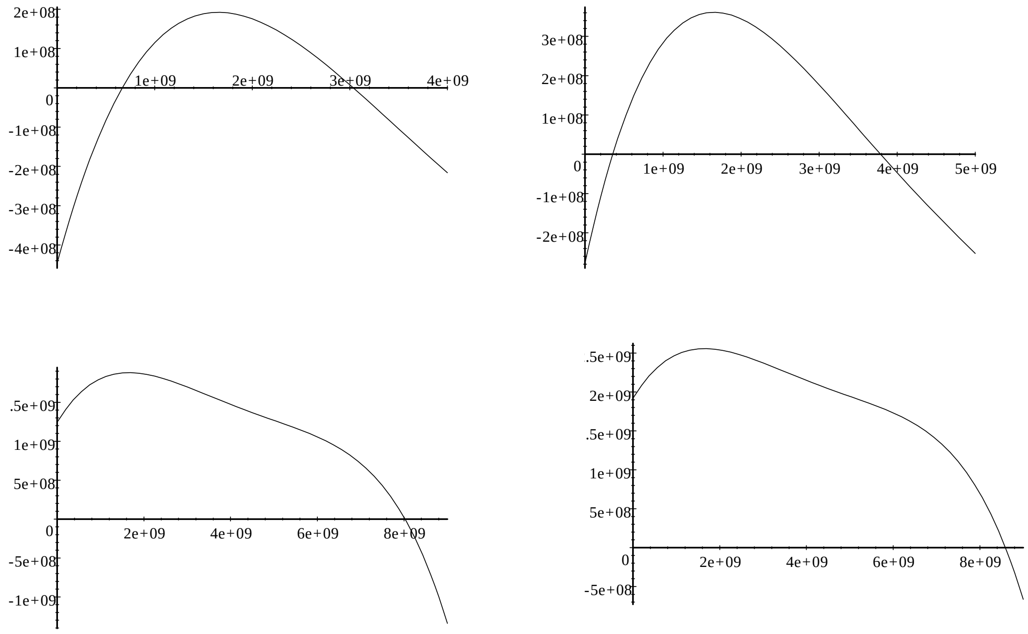
Figure A7 Population Forecast Lower Income Countries, (2)



Forecast: LIF
Actual: LI
Forecast sample: 1960 2100
Adjusted sample: 1962 2100
Included observations: 40

Root Mean Squared Error	1443878.
Mean Absolute Error	1154588.
Mean Abs. Percent Error	0.068678
Theil Inequality Coefficient	0.000409
Bias Proportion	0.000226
Variance Proportion	0.000627
Covariance Proportion	0.999147

Figure A8



Population for the world shifts upward according to Foley's model. In 1960, 1970, 2060, and 2100 the population levels are (predicted to be) 3.06, 3.68, 8.03, 8.59 billion people.

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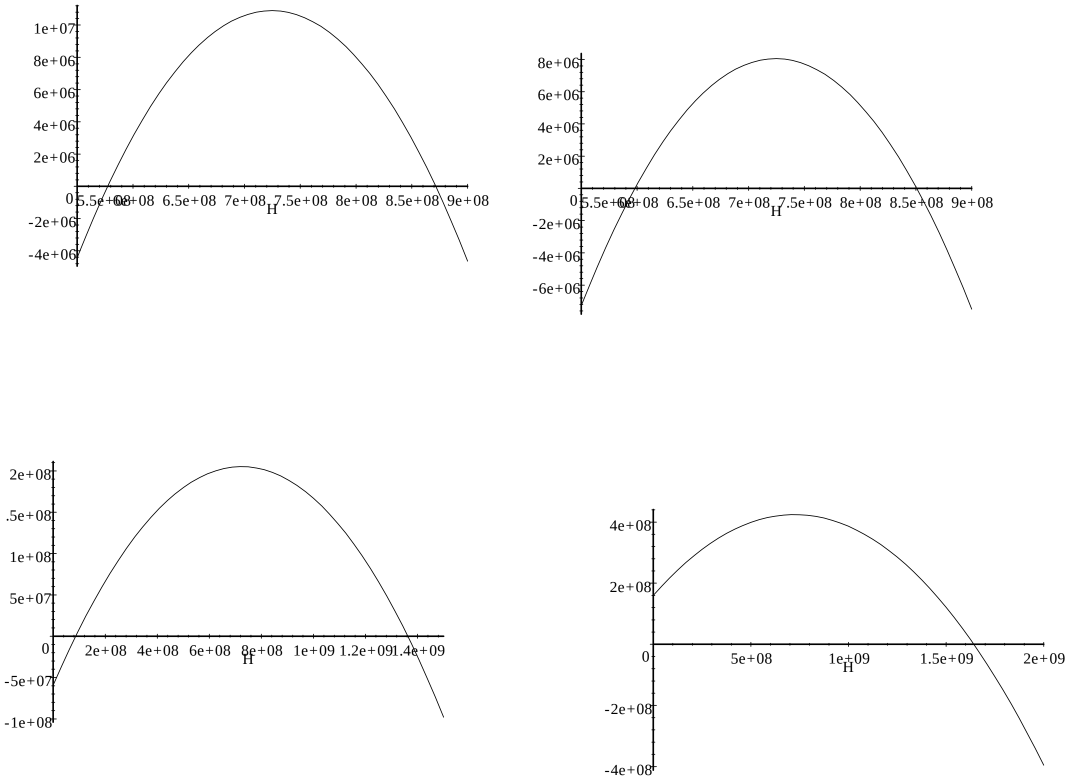
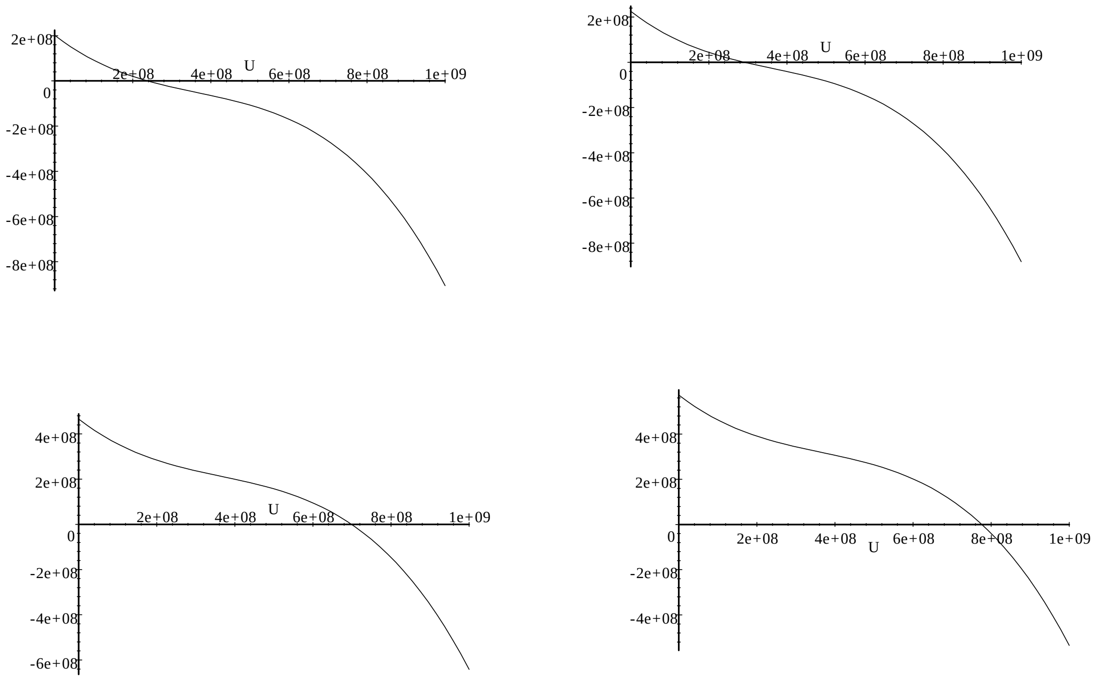
Figure A9



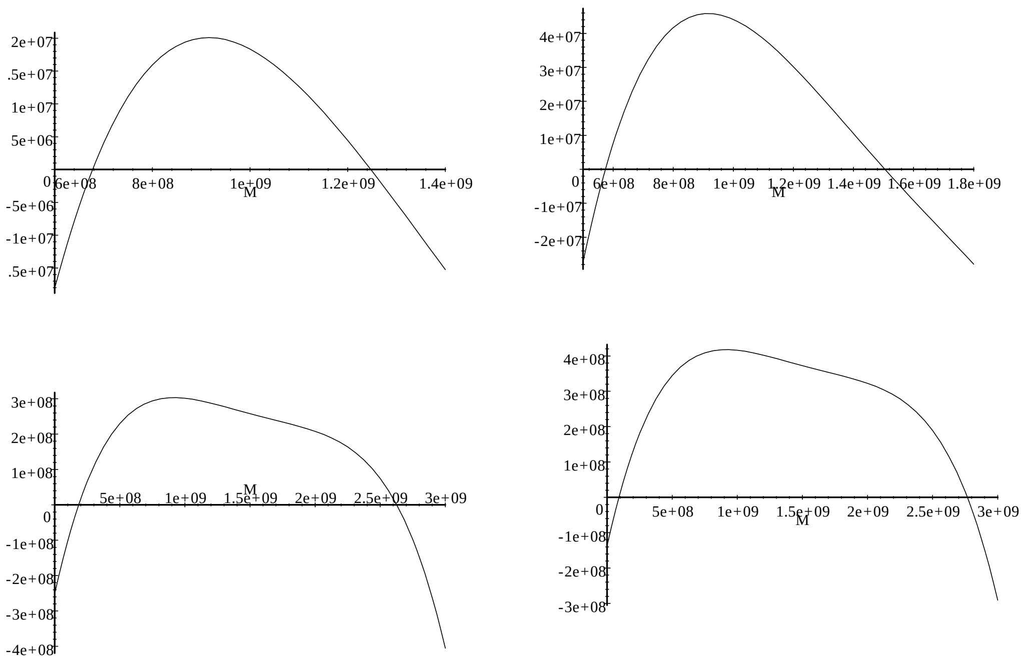
The population growth curve for the high-income countries has moved up and has become broader. The population in 1960, 1970, 2060 and 2100 is (predicted to be) .6772, .758, 1.37, 1.65 billion.

Figure A10



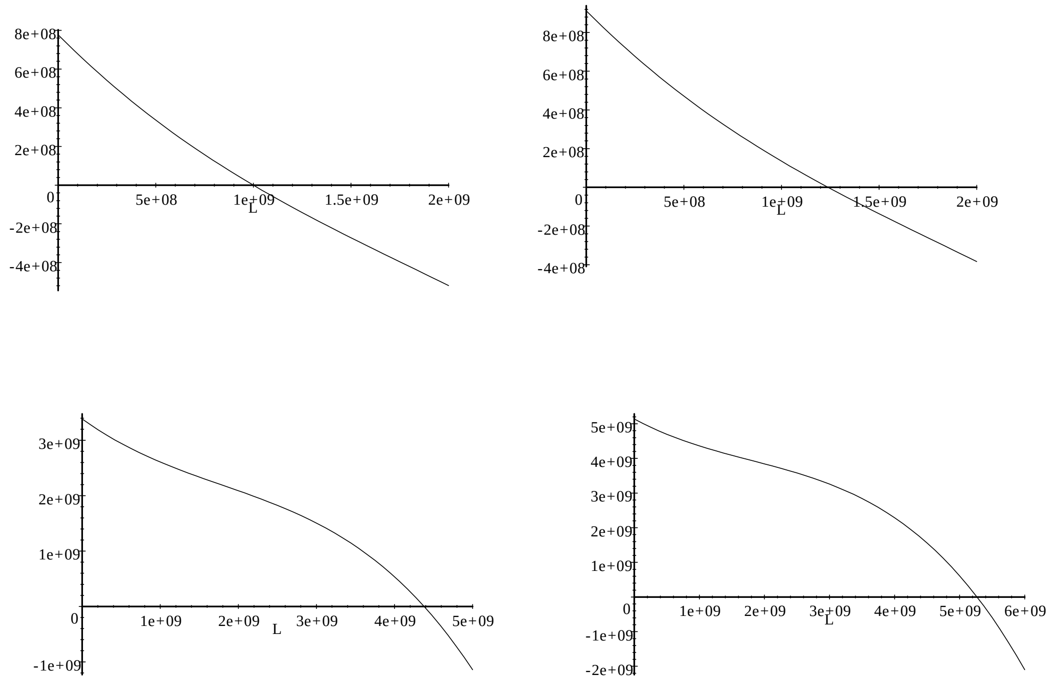
The population dynamics of upper middle-income countries has only one moving steady state shifting to the right. The population in 1960, 1970, 2060 and 2100 is (predicted to be) .236, .294, .784, and .931 billion.

Figure A11



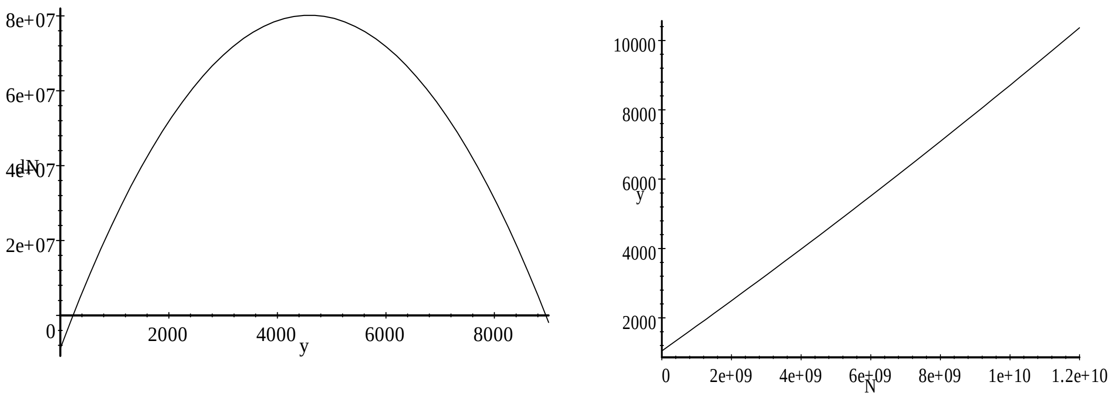
The population dynamics of the lower-middle-income countries has a stable and an unstable steady state. The whole curve shifts up over time. The population in 1960, 1870, 2060 and 2100 is (predicted to be) 1.1, 1.35, 2.65, 2.76 billion.

Figure A12



For low-income countries there is only one moving steady state shifting to the right. The population in 1960, 1870, 2060 and 2100 is (predicted to be) 1.01, 1.27, 4.4, 5.31 billion.

Figure A13: Estimation for aggregate world data



For aggregate world data we find increasing returns to scale, an unstable Malthusian and a stable Smithian steady state and no time trends. Per capita income for 2000 is y= 4750.

Figure A14: High income countries in 1960, 2000, 2060, 2100

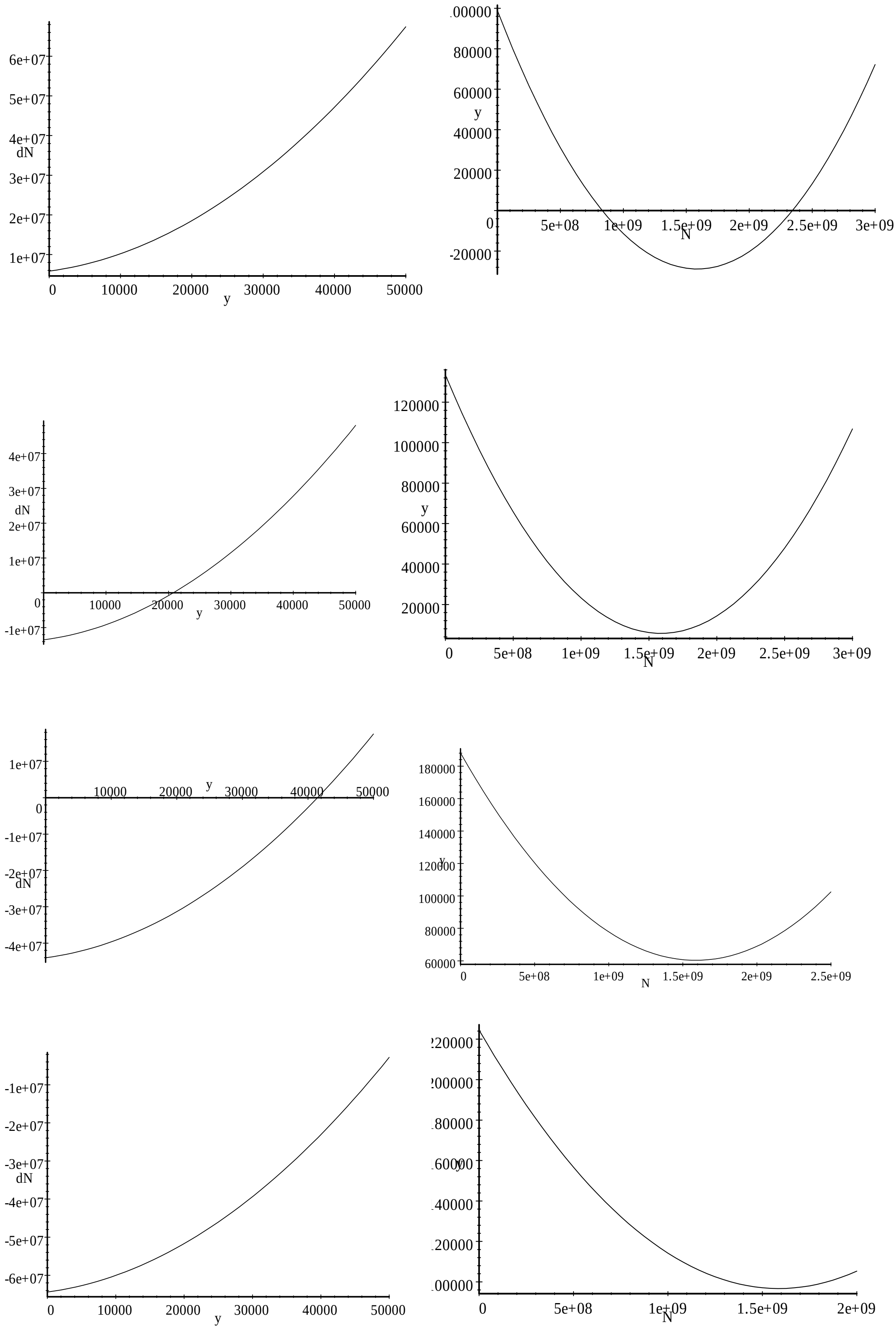


Figure A15: Upper middle income countries in 1960, 2000, 2060 and 2100

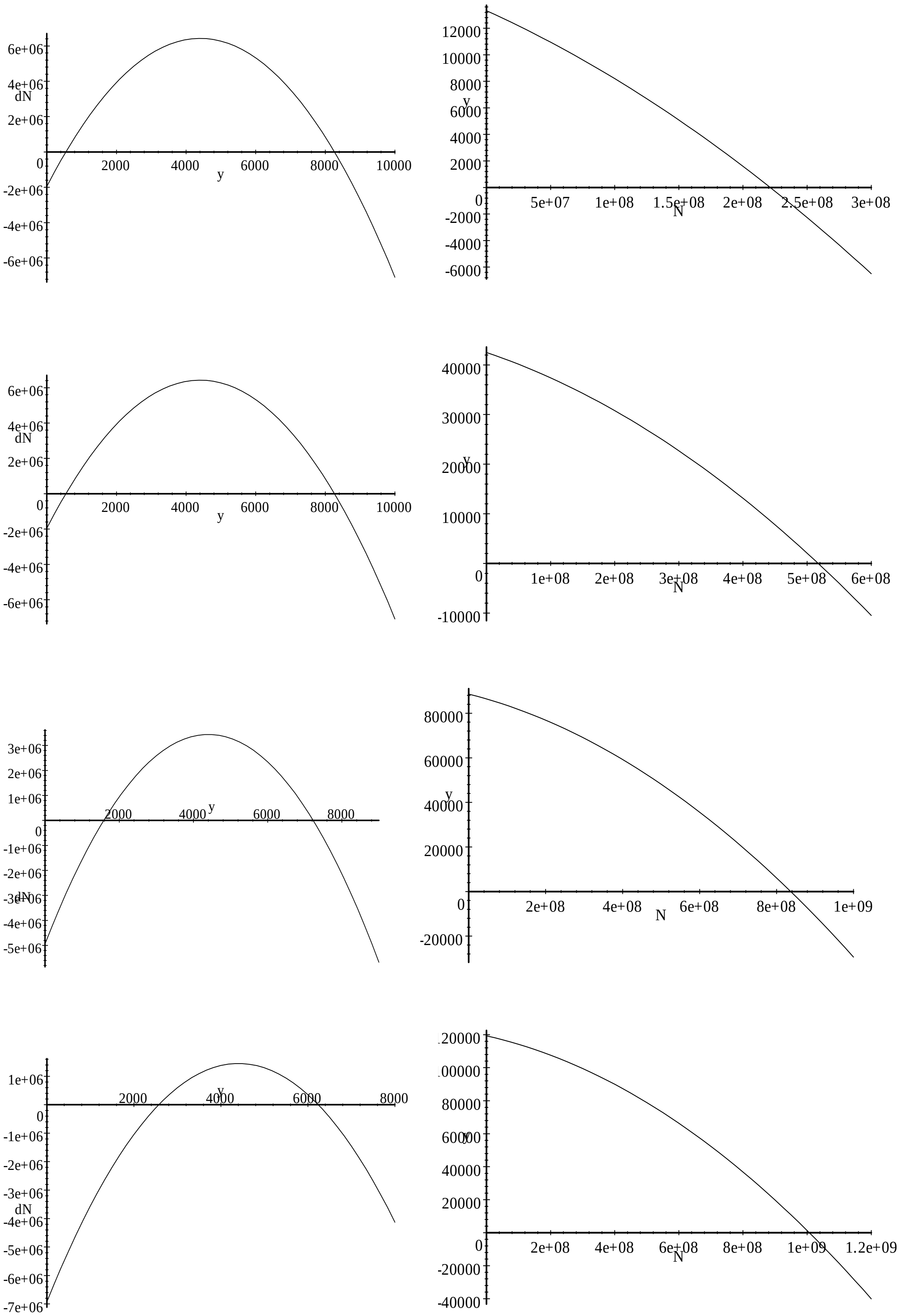


Figure A16: Lower middle income countries 1960, 2000

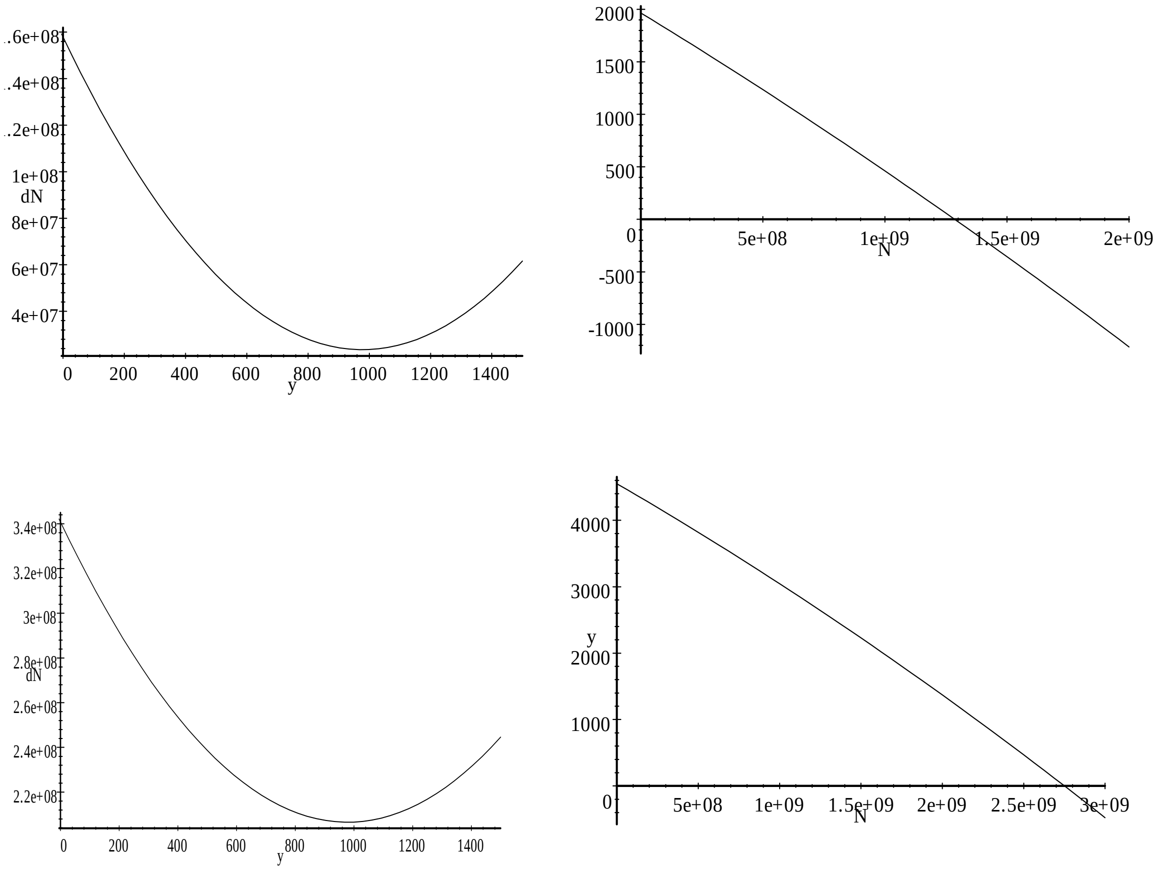


Figure A17: Low income countries in 1960, 2000, 2060, 2100.

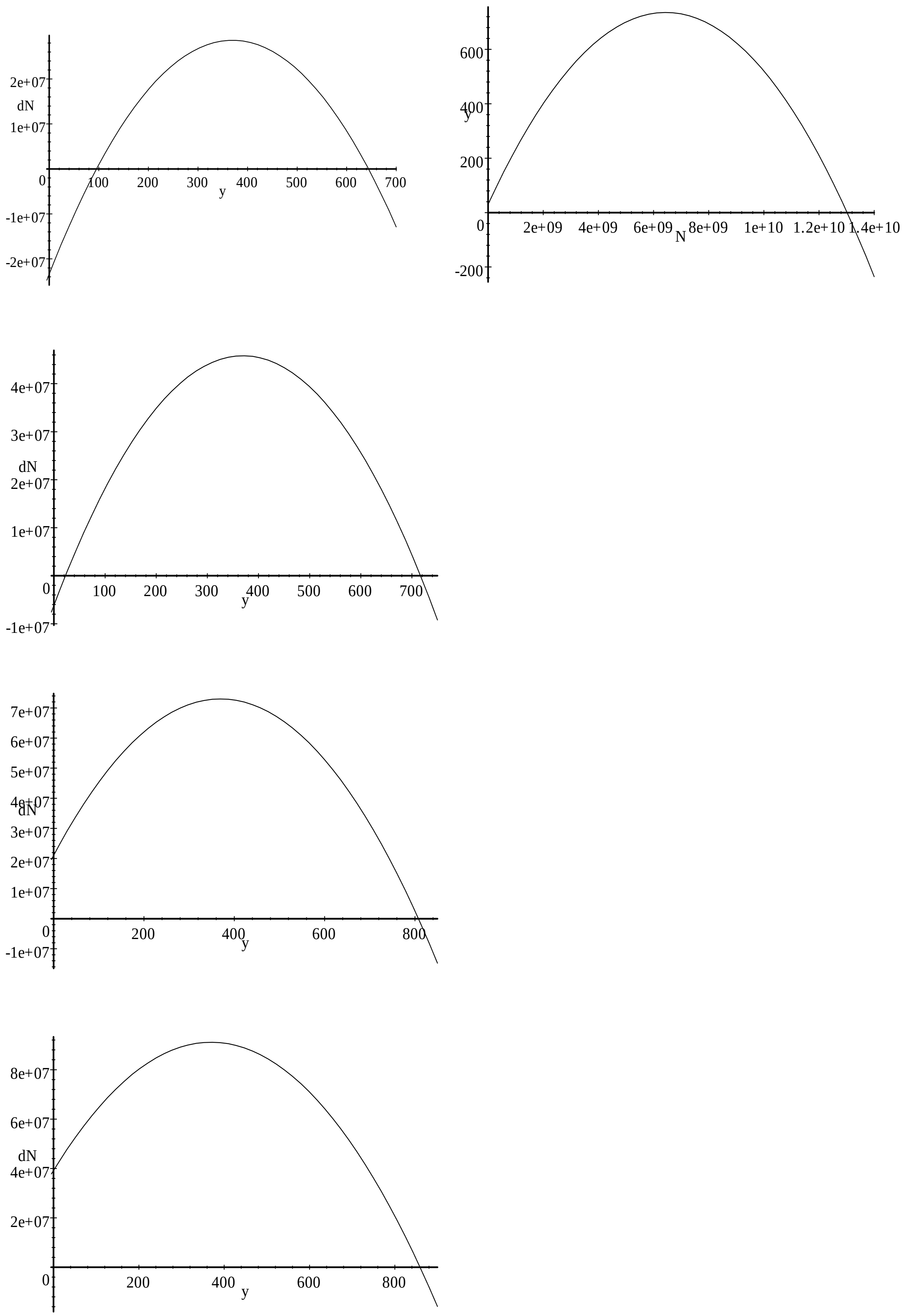


Figure A18: Sub-Saharan Africa 1960, 2000, 2060

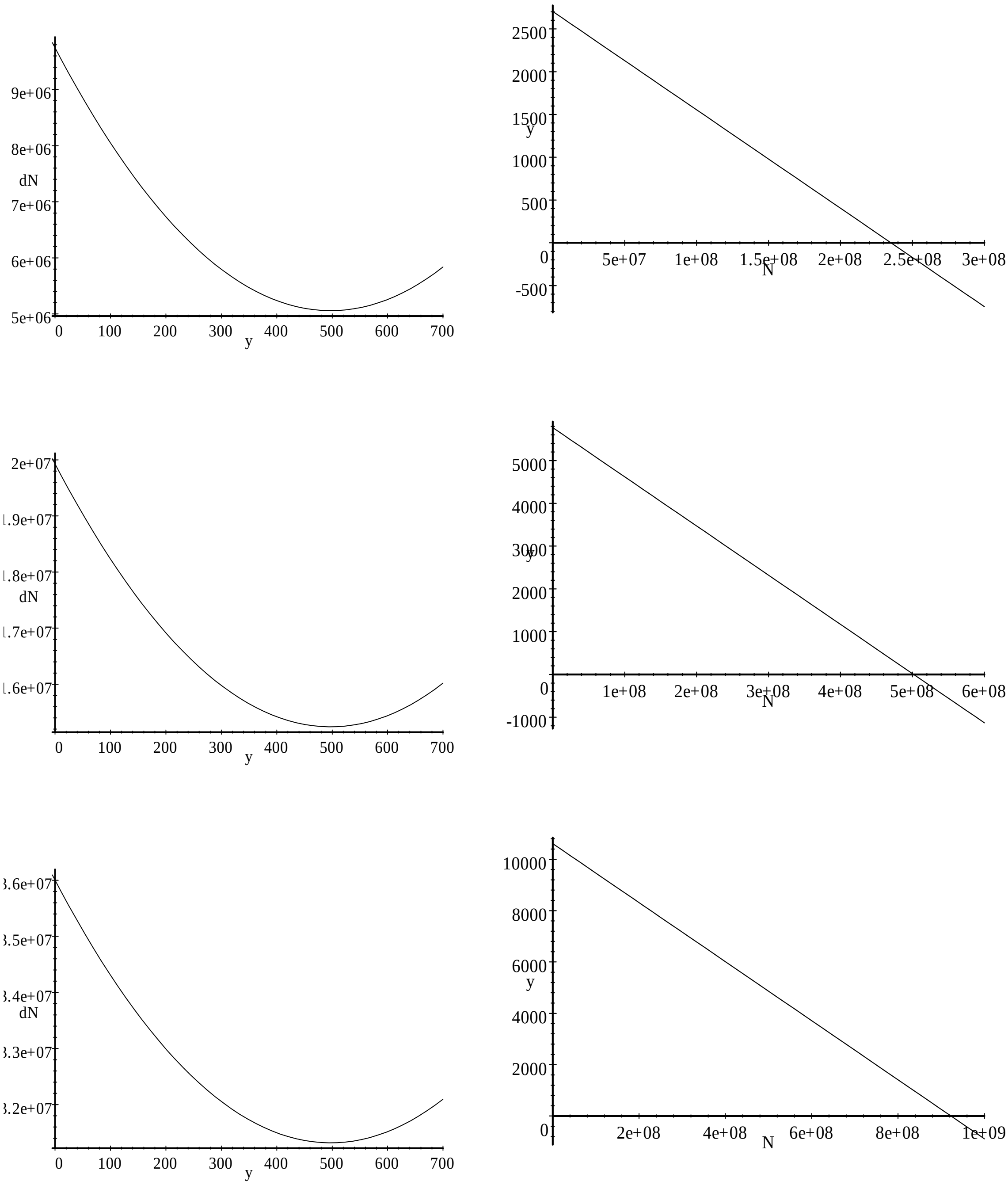


Figure A19: GDP Per Capita (constant 1995 US dollars) – *restricted dataset*

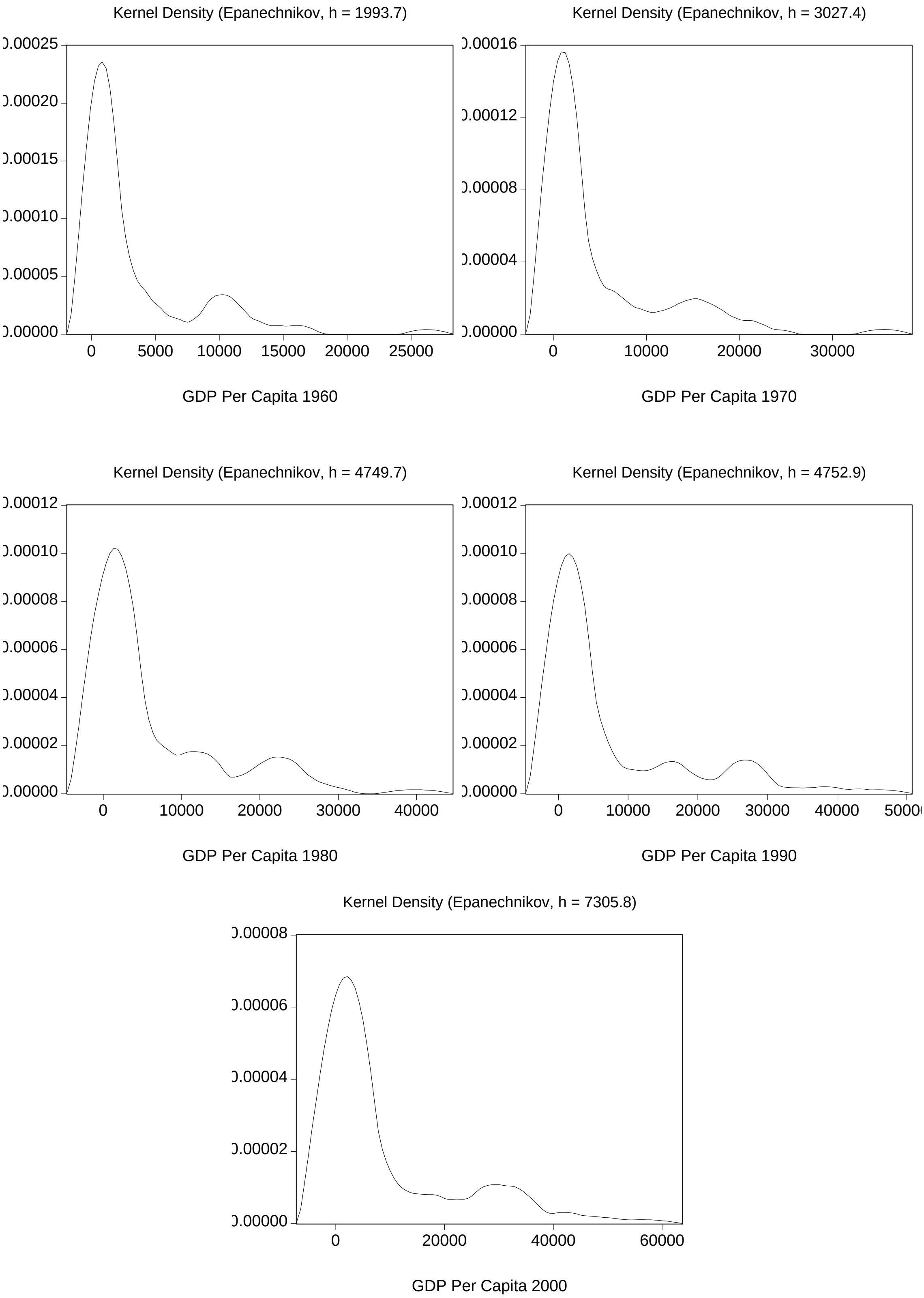


Figure A20: GDP Per Capita (constant 1995 US dollars) – *unrestricted dataset*

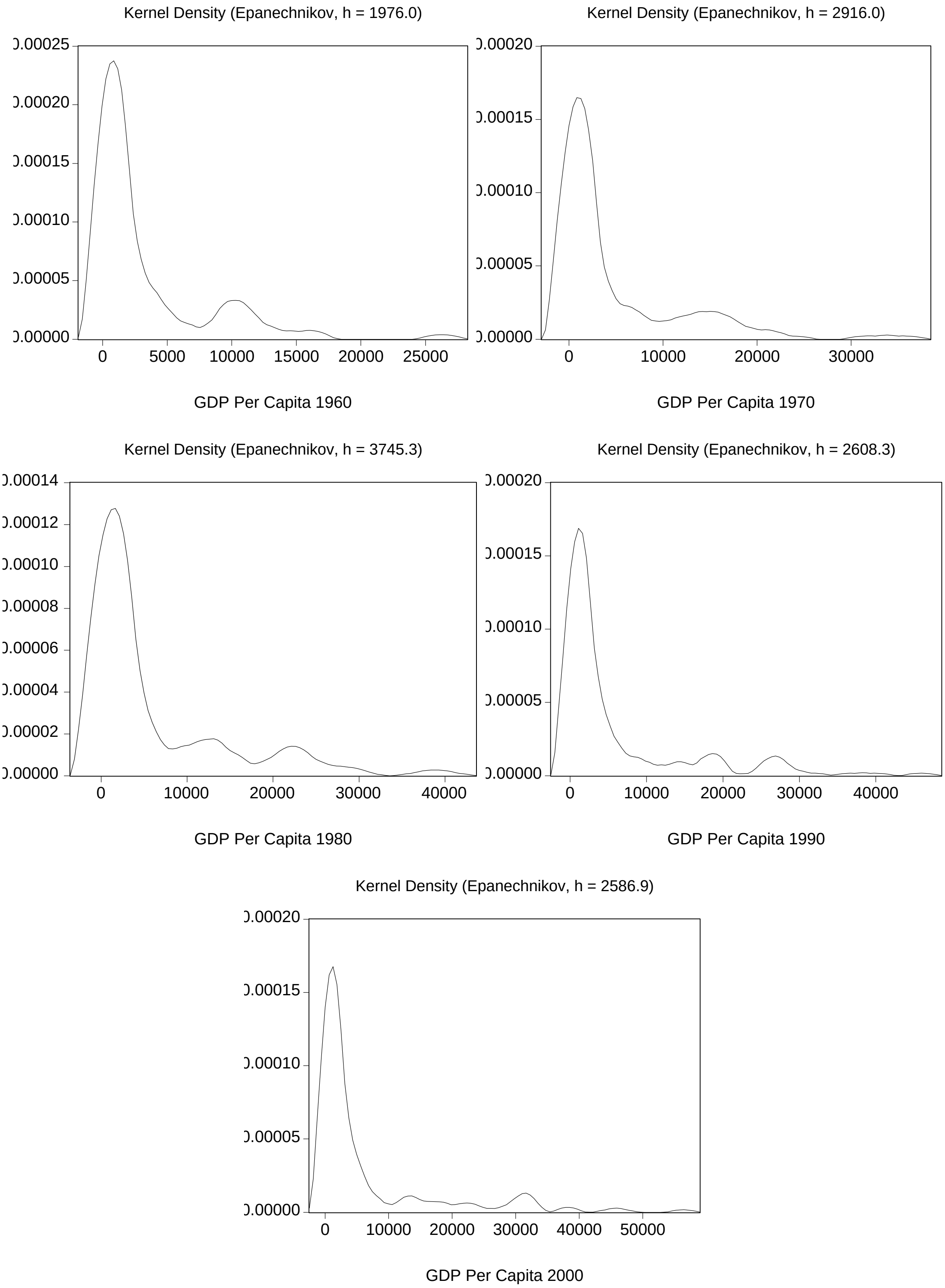
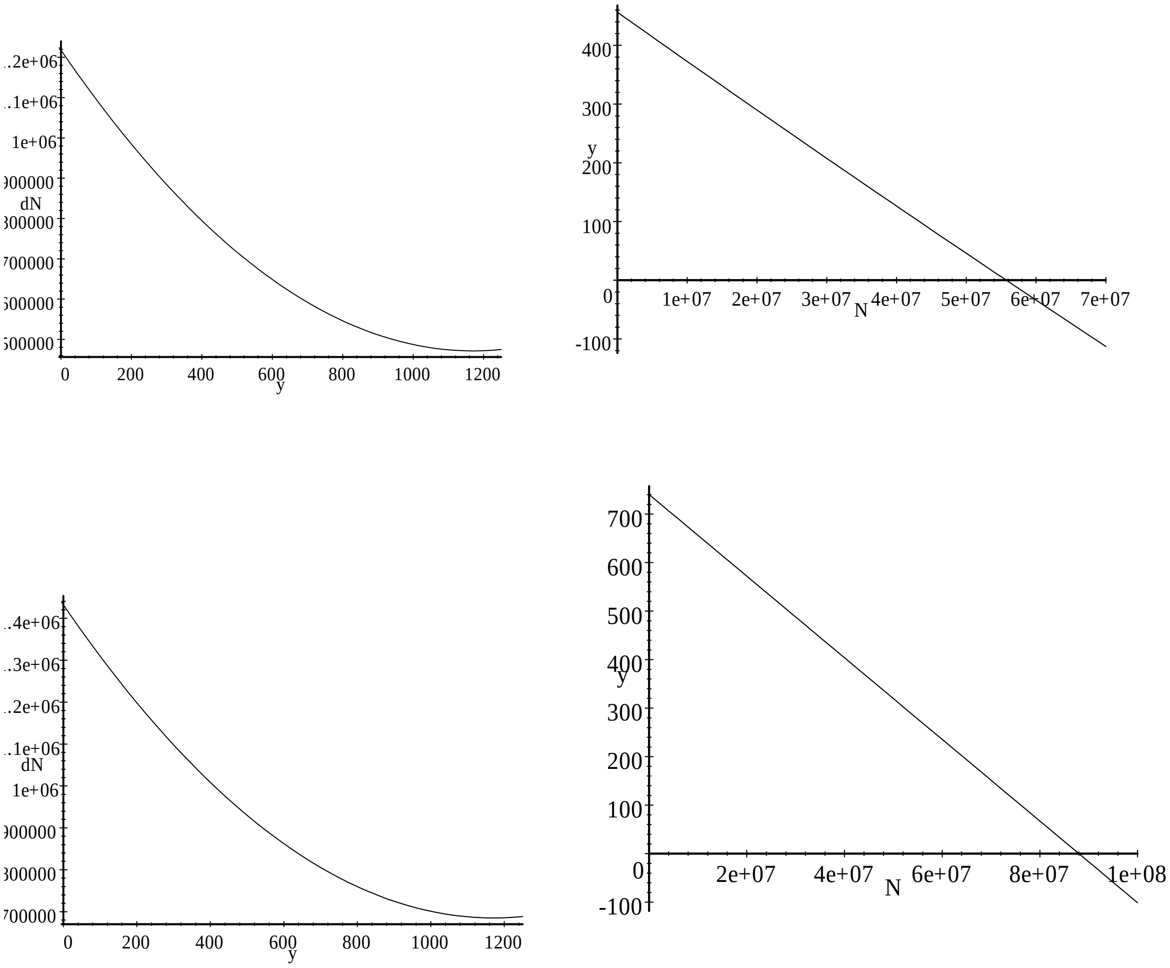


Figure A21: The U1200 group for 1960 and 2000



The plot of the panel estimate result of Foley’s model, for countries with GDP per capita below $1200 for the years 1960 and 2000, shows Smithian population behaviour and decreasing returns to scale. The panel average values are y = $475, N = 38mln., and, for the years 2000-2002, y=$494, N=52.5mln .

Figure A22: Distribution of coefficients of time shifts in the production function of the u1200 countries